

Small Lorentz violations in quantum gravity: do they lead to unacceptably large effects?

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Abstract

We discuss the applicability of the argument of Collins, Pérez, Sudarsky, Urrutia and Vucetich to loop quantum gravity. This argument suggests that Lorentz violations, even ones that only manifest themselves at energies close to the Planck scale, have significant observational consequences at low energies when one considers perturbative quantum field theory and renormalization. We show that non-perturbative treatments like those of loop quantum gravity may generate deviations of Lorentz invariance of a different type than those considered by Collins et al. that do not necessarily imply observational consequences at low energy.

Over time, the notion that quantum gravity may impose some level of discreteness to space-time at the Planck scale, has been put forward in many contexts (see [1] for a reviews). In loop quantum gravity (LQG) and other approaches it is many times —though not universally— asserted that there is a fundamental level of discreteness. For instance, discretized models of spherically symmetric space-times seem to impose a Planck scale lattice at a fundamental level [2]. So do models of parameterized field theory [3]. In spin foam approaches it is sometimes assumed that one will not take the limit in which the spacing in the foam goes to zero [4] but rather look for coarse graining of the physical quantities at large distances compared to the Planck scale. The presence of a fundamental discreteness does not necessarily imply that there exist violations of the local Lorentz invariance. For instance, many spin foam models implement local Lorentz invariance exactly. However, some of the models of discreteness seem to violate local Lorentz invariance. Lorentz violating theories have also received recent attention in relation to Hořava’s proposal of gravity at the Lifshitz point [5]

In such contexts an argument due to Collins, Pérez, Sudarsky, Urrutia and Vucetich[6, 7] seems to be quite relevant. These authors have studied what happens to perturbative quantum field theories that violate Lorentz invariance. They notice that when one renormalizes such theories even minute Lorentz violations that manifest themselves only at the Planck scale become quite relevant. When one regularizes and renormalizes there are certain terms that would be divergent that just happen to be zero due to Lorentz invariance. When the latter is broken, even by amounts that are very small at low energies, the regularization and renormalization process can render results that violate Lorentz invariance in amounts not suppressed by the energy. These effects are large enough to render the resulting theories experimentally not viable.

Should such an argument imply that the Lorentz violations that may arise in LQG due to its discreteness render the theory not viable experimentally? There is a bit of a gap in the argument. The original argument talks about building perturbative quantum field theories that start being Lorentz non-invariant and relies on the use of renormalization. But LQG is non-perturbative and in most calculations one does not need to renormalize since the theory is finite. Does this imply that the argument does not apply in this case? Not necessarily. LQG aspires to make contact for low energies with ordinary perturbative quantum field theory. How is such a contact going to accommodate potential Lorentz violations without

running into the problems pointed out by Collins et al.?

To illustrate the problem, we will consider a simplified situation, discussed by Collins et al. [7], the case of a Yukawa interaction of a scalar field and a Fermion. However, we will not introduce by hand Lorentz violating terms that go as E/E_{Planck} . We will put the theory on a lattice and we will assume the lattice spacing is small compared to particle physics lengths but large or comparable to the Planck scale, and that will be our source of Lorentz non-invariance. This is the situation one faces, for instance, in spherical models of LQG, which leads to similar types of propagators as those of the model we consider. We will not take the limit in which the lattice spacing goes to zero (if we did we would reproduce the results of Collins et al. without Lorentz violating terms, just with a different regularization, as they used Pauli–Villars to regularize).

On the lattice the propagators differ from the continuum ones. The one for the Fermion is,

$$S(k) = \frac{m - i \sum_{j=1}^3 \gamma^j a^{-1} \sin(ak_j) - i\gamma^0 (ba)^{-1} \sin(bak_0)}{m^2 + a^{-2} \sum_{j=1}^3 (2 - 2 \cos^2(ak_j)) + (ba)^{-2} (2 - 2 \cos^2(bak_0))}. \quad (1)$$

We are considering the Euclidean case as is common in lattice field theory and we are allowing different lattice spacings in space a and time, ba with b an arbitrary factor that tends to one in the continuum limit; unequal spacings in space and time might occur in LQG in the canonical approach). For the scalar we have ,

$$G(k, m) = \frac{1}{m^2 + a^{-2} \sum_{j=1}^3 (2 - 2 \cos^2(ak_j)) + (ba)^{-2} (2 - 2 \cos^2(bak_0))}, \quad (2)$$

The derivation of these propagators can be seen in reference [8]. In the limit $a \rightarrow 0$ one recovers the usual expressions for the propagator. If one computes the correlation length between two lattice sites \vec{n}_1 and \vec{n}_2 , $\gamma(\vec{n}_1, \vec{n}_2)$, given by taking the Fourier transform of $G(k, m)$ it will in general go as $G \sim \exp(-t/\gamma(\vec{n}_1, \vec{n}_2))$ with t the distance between the two lattice site will in general depend on the choice of lattice sites. However, in the continuum limit it becomes isotropic, taking the quotient of two γ 's in arbitrary directions $\frac{\gamma(\vec{n}_1, \vec{n}_2)}{\gamma(\vec{n}_1, \vec{n}_3)} \sim 1 + O(m^2 a^2)$.

This indicates that the lattice treatment restores the invariances of the continuum for finite values of a in a continuous way. It gives a hint that in general one will not have large departures from the continuum for small lattice spacings, as we will see.

The sum over irreducible one particle two-point graphs, called the self-energy $\Pi(p)$, yields the corrections to the propagation of the scalar field. Following Collins et al., we study its

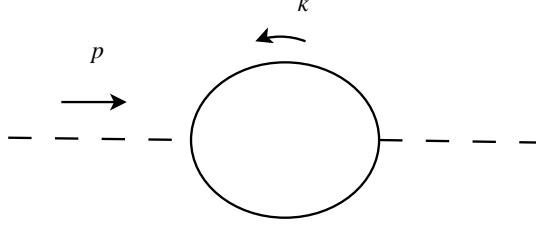


FIG. 1: Feynman diagram for the lowest order self-energy. The diagram includes an unbound integral in k that can, for energy dependent Lorentz violations, lead to large observable effects even if the Lorentz violations are small for low energies.

value for momenta and mass much smaller than a cutoff Λ (that in the lattice case will be related to the inverse of a) and to one loop order, as shown in figure 1, which we denote by $\Pi_1(p)$. With no cutoff the diagram is quadratically divergent. One has that,

$$\Pi_1(p) = A + p^2 B + p^\mu p^\nu W_\mu W_\nu \tilde{\xi} + \Pi^{(LI)}(p^2) + O(p^4/\Lambda^2). \quad (3)$$

with W_μ a timelike four vector. In the continuum the first two terms diverge and the divergence is absorbed in a redefinition of the mass and wavefunction. The fourth term is Lorentz invariant and finite. The fifth term is Lorentz violating but suppressed. The third term may contain unsuppressed Lorentz violations. To obtain the third term we take the self-energy and differentiate it twice, $\xi = \frac{\partial^2 \Pi_1(p)}{\partial (p^0)^2} - \frac{\partial^2 \Pi_1(p)}{\partial (p^1)^2}$, and evaluate it at $p = 0$. Such a quantity obviously vanishes for a Lorentz invariant theory (differences in signs with Collins et al. are due to us working in the Euclidean case). Again following Collins et al. in the continuum one has,

$$\tilde{\xi} = -\frac{ig^2}{\pi^4} \int d^4 k \frac{[-(k^0)^2 + (k^1)^2](k^2 - 3m^2)}{(k^2 + m^2)^4} \quad (4)$$

and in the lattice the analogous expression is given by,

$$\begin{aligned} \tilde{\xi}_a = & \int_{-\pi/(ba)}^{\pi/(ba)} dk_0 \int_{-\pi/a}^{\pi/a} dk_1 \int_{-\pi/a}^{\pi/a} dk_2 \int_{-\pi/a}^{\pi/a} dk_3 \left(\frac{\sum_j \sin^2(ak_j)}{a^2} + \frac{\sin^2(bak_0)}{b^2 a^2} + m^2 \right)^{-4} \\ & \times \left[16 \left(\frac{\sum_j \sin^2(ak_j)}{a^2} + \frac{\sin^2(bak_0)}{b^2 a^2} + m^2 \right) \left(\frac{\sin^2(2bak_0)}{4b^2 a^2} - \frac{\sin^2(2ak_1)}{4a^2} \right) \right. \\ & + 4 \left(\sin^2(bak_0) - \sin^2(ak_1) \right) \left(\frac{\sum_j \sin^2(ak_j)}{a^2} + \frac{\sin^2(ak_0)}{b^2 a^2} + m^2 \right)^2 \\ & \left. + 32 \left(\frac{\sin^2(2bak_0)}{4b^2 a^2} - \frac{\sin^2(2ak_1)}{4a^2} \right) \left(m^2 - \frac{\sum_j \sin^2(ak_j)}{a^2} - \frac{\sin^2(bak_0)}{b^2 a^2} \right) \right] \end{aligned}$$

$$-8 (\cos (2b a k_0) - \cos (2a k_1)) \left(m^4 - \left(\frac{\sum_j \sin^2 (a k_j)}{a^2} + \frac{\sin^2 (b a k_0)}{b^2 a^2} \right)^2 \right) \Bigg] \quad (5)$$

where one should strictly have a sum over k but it can be approximated with an integral with a cutoff of order $1/a$, that is, each spatial integral goes from $-\pi/a$ to π/a and similarly the time integral goes from $-\pi/(b a)$ to $\pi/(b a)$.

Collins et al. showed that in the continuum that $\tilde{\xi}$ is finite in the limit when the cutoff goes to infinity if one has propagators that violate Lorentz invariance even if the violation occurs only at very high energies. That means one has Lorentz violations unsuppressed by the energy. Let us study the situation in the lattice. If one has a finite lattice spacing and computes the integral (7), if one takes the same spacing in space and in time, the integral vanishes by symmetry reasons. If one takes different spacings in space and time (but such that the limit is isotropic, for instance $b = 1 + \mu a$ with μ a parameter with dimensions of mass) one gets a finite result, that in the limit,

$$\lim_{a \rightarrow 0} \xi_a = 0, \quad (6)$$

and if one expands the finite result in powers of a , one gets that the leading contribution is of the order of $a \mu$. That is, there is a departure from the Lorentz invariant case, but if the lattice spacing is small the departure is small unlike the result of Collins et al. It is important to notice the order in which limits are taken. If one takes $a \rightarrow 0$ before computing the integral one is back to the continuum Lorentz-invariant calculation.

The central message seems to be that the types of approaches advocated in LQG, where one considers lattice spacings that are small but non-vanishing, may not lead to the type of Lorentz-violating contributions that were considered by Collins et al.

The model we are studying has some imperfections as a complete illustration of the situation one faces in LQG. First of all, we have not done a LQG treatment of the model, but only mimicked the situation using a lattice. Second, we used a space-time lattice. At first this may appear quite problematic: in LQG one may work in a canonical framework. Should we not have used a lattice only in space? The propagators with a spatial lattice and continuum time would have denominators $(p^0)^2 + \sum_{i=1}^3 \sin(ap^i)^2/a^2$. The integral in the p^0 variable is now computed on an unbound domain and leads again to the types of effects Collins et al. pointed out. Is this a problem? We do not necessarily think so. LQG being a canonical approach, it has the “problem of time”. That is, the parameter associated with

the orbits of the Hamiltonian constraint, which is continuous, is not to be identified as time. One possible way out of the problem of time is to do a relational treatment where one picks one of the variables as a quantum clock [9]. Such clocks may have continuous spectrum, but due to their quantum nature they have uncertainties associated with them. That situation is not modeled well by the discussion of the current paper, but we would like to argue that the dispersion in the clock variable may cure the divergences introduced by the p^0 terms and eliminates any large departure from Lorentz invariance. Let us sketch how this works. In quantum gravity time will be given by an operator $T(x^0)$. The propagator will be the ordinary one multiplied a couple of probability distributions,

$$D(T, \vec{x}, T', \vec{x}') = \int_{-\infty}^{\infty} dt dt' D(t, \vec{x}, t', \vec{x}') \mathcal{P}(t, T) \mathcal{P}(t', T'), \quad (7)$$

where the probability $\mathcal{P}(t, T)$ is the probability that the real clock variable takes a value T when the ideal parameter is t . In situations where there is a well defined notion of time one expects $\mathcal{P}(t, T)$ to be a peaked function close to a Dirac delta (the ideal and real time correlate well). The extra integral of the peaked function cuts off the high frequency contributions in zeroth component of the momentum in the propagator. The others are cut off because of the spatial lattice. Therefore the resulting space-time propagator is an ordinary (non distributional) function. As an example if one takes \mathcal{P} to be a step function regularization of the delta of width σ centered in t and t' , one gets,

$$D(T, \vec{x}, T', \vec{x}') = \int_{-\pi/a}^{\pi/a} d^3p \frac{e^{i\vec{p}\cdot\vec{x}} \sin^2(\omega_a \sigma)}{2\omega_a \omega_a^2 \sigma^2} e^{-i\omega_a |T-T'|}, \quad (8)$$

where $\omega_a = \sqrt{m^2 + \sum_j \frac{\sin^2(ap_j)}{a^2}}$ and it leads to finite expressions for the Feynman diagrams. In the limit $a \rightarrow 0$, $\sigma \rightarrow 0$ one obtains the usual propagator and, Lorentz invariance of the propagator is recovered in the limit. In usual quantum gravity scenarios σ is proportional to some power of the Planck length and grows with time. That means that both the spatial and temporal effects would both go to zero if one were to take the Planck length to zero. This ensures some uniformity in the limit for what are two very different effects. However, it should be emphasized that the above calculation is just a sketch, with many implications yet to be fleshed out, and at the present level of development one cannot conclusively state that the limit will behave as in the lattice example we discussed in detail. It should also be added that in a field theory like general relativity one not only expects to include physical clocks to measure time but also physical measuring rods to measure space (a brief discussion

is in [10]). If one does that one will produce smearings similar to the one we just discussed in the other components of the momentum and this will further help avoid large Lorentz violations. An analogous situation develops in attempts to regulate theories using Lorentz-violating non-commutative and fuzzy space [11].

It could also be criticized that we worked in the Euclidean theory as is commonly done in lattice treatments, something one does not expect to do in the case of LQG. The truth is that any regularization procedure that violates Lorentz invariance but preserves the invariance at the dominant order in the ultraviolet regime will behave like the calculation we carried out. We chose the Euclidean lattice as an example, but this is not central to the main argument. For instance, as an example of a regularization of the type mentioned, one could have used a Lorentz violating Pauli–Villars regularization. The usual (Lorentz invariant) Pauli–Villars regularization consists in including an additional particle with mass M ,

$$\frac{1}{-k_0^2 + \vec{k}^2 + m^2} - \frac{1}{-k_0^2 + \vec{k}^2 + M^2} \quad (9)$$

and in the limit of large M this regularizes the theory because the propagator instead of going as $1/k^2$ goes as $1/k^4$. A Lorentz violating Pauli–Villars regularization of the propagator could be,

$$\frac{1}{-k_0^2 + \vec{k}^2 + m^2} - \frac{1}{-k_0^2 + \vec{k}^2 + \frac{m^4}{\vec{k}^2 + M^2} + M^2}. \quad (10)$$

This still regularizes the theory and the Lorentz violating terms go to zero in the limit in which one removes the regulator. One can check that computing ξ one gets a small contribution of the order m^4/M^4 . Although people commonly perform a Wick rotation when working with Pauli–Villars regularizations as a matter of convenience, it is not necessary to work that way, one can work in an entirely Lorentzian fashion. The Pauli–Villars example is problematic in that outside of the limit the resulting quantum field theory is ill defined. One can however think of other regularization schemes that yield a well defined quantum field theory even when the regulator is not removed. Also, the Pauli–Villars example suggests that having unbound integrals in k_0 does not automatically lead to the problems presented in Collins et al. Another example of this is given in the paper of Reyes, Urrutia and Vergara [12]. But as our own example shows, one cannot take any lattice regularization, certain conditions of regularity had to be met. This is particularly relevant in the context of gravity where irregular lattices are commonly used. We have not studied that case in detail up to now.

In hindsight, the findings of this paper are not surprising. It is well known that *any* regularization procedure produces, before taking the limit, propagators that depend on a (dimensionful) parameter and that are finite. Outside of the limit, all regularizations produce results that are close to the continuum limit plus terms that are absorbed in redefinitions of the fundamental parameters. The important point is there exists continuity in the regularization parameter. There is abundant literature (e.g. [13]) on what happens outside of the limit and it is well known that new physics arises. Analyticity properties change at high energy. But, with the hypotheses mentioned above, there are no large violations of Lorentz invariance. All this is in line with what one would expect in a context like LQG.

An additional point is that one should be extra careful when considering expansions of quantities. For instance if one had considered the propagator in the lattice and expanded in the limit of small lattice spacing a , and only kept the leading terms, instead of having trigonometric functions one would have powers of the momentum. Such terms would lead to large contributions to ξ since they do have the form considered by Collins et al. However it is clear from the discussion of the present paper that analyzing things in such a way is incorrect. We had carried out analyses of that type in unpublished work [14]. It would also be the case if one attempted to apply the analysis of Collins et al. to the polymer propagators computed in [15].

Summarizing, assuming that —as expected— LQG could lead to a finite divergence-free theory whose low energy approximation for matter fields will be of the form of a regularized version of ordinary quantum field theories, small potential departures from Lorentz invariance of the exact theory do not necessarily spoil the Lorentz invariance at low energy. The main difference with the Collins et al. example is that they start with Lorentz violating propagators that do not have the form that would result from a regularization of the matter fields, something many expect LQG will provide.

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